CHEAT SHEETS HERO

Discrete Mathematics Cheatsheet

A comprehensive cheat sheet covering key concepts in Discrete Mathematics, including set theory, logic, relations, graph theory, and combinatorics. This serves as a quick reference for definitions, formulas, and common problem-solving techniques.

Set Theory

Basic Definitions

Set	A well-defined collection of distinct objects, considered as an object in its own right.
Element	An object in a set. Denoted by ∈ (e.g., x ∈ A means x is an element of set A).
Subset	A set A is a subset of B (A ⊆ B) if every element of A is also in B.
Proper Subset	A set A is a proper subset of B (A ⊂ B) if A ⊆ B and A ≠ B.
Universal Set (U)	The set containing all elements under consideration.
Empty Set (∅)	The set containing no elements. Also denoted by {}.

Set Operations

Union (∪)	A ∪ B = {x | x ∈ A or x ∈ B}
Intersection (∩)	A ∩ B = {x | x ∈ A and x ∈ B}
Difference (-)	A - B = {x | x ∈ A and x ∉ B}
Complement (A')	A' = {x | x ∈ U and x ∉ A}
Symmetric Difference (⊕)	A ⊕ B = (A - B) ∪ (B - A)
Cartesian Product (×)	A × B = {(a, b) | a ∈ A and b ∈ B}

Set Identities

Identity Laws:
A ∪ ∅ = A
A ∩ U = A

Domination Laws:
A ∪ U = U
A ∩ ∅ = ∅

Idempotent Laws:
A ∪ A = A
A ∩ A = A

Complementation Law:
(A')' = A

Commutative Laws:
A ∪ B = B ∪ A
A ∩ B = B ∩ A

Associative Laws:
A ∪ (B ∪ C) = (A ∪ B) ∪ C
A ∩ (B ∩ C) = (A ∩ B) ∩ C

Logic

Propositional Logic

Proposition	A declarative statement that is either true or false, but not both.
Conjunction (∧)	p ∧ q is true if both p and q are true; otherwise, it is false.
Disjunction (∨)	p ∨ q is true if either p or q (or both) are true; it is false only if both are false.
Negation (¬)	¬p is true if p is false, and false if p is true.
Implication (→)	p → q is false only when p is true and q is false; otherwise, it is true. Also called a conditional statement.
Biconditional (↔)	p ↔ q is true if p and q have the same truth value (both true or both false).

Logical Equivalences

De Morgan's Laws:
¬(p ∧ q) ≡ ¬p ∨ ¬q
¬(p ∨ q) ≡ ¬p ∧ ¬q

Distributive Laws:
p ∧ (q ∨ r) ≡ (p ∧ q) ∨ (p ∧ r)
p ∨ (q ∧ r) ≡ (p ∨ q) ∧ (p ∨ r)

Implication Equivalence:
p → q ≡ ¬p ∨ q

Biconditional Equivalence:
p ↔ q ≡ (p → q) ∧ (q → p)

Commutative Laws:
p ∧ q ≡ q ∧ p
p ∨ q ≡ q ∨ p

Associative Laws:
(p ∧ q) ∧ r ≡ p ∧ (q ∧ r)
(p ∨ q) ∨ r ≡ p ∨ (q ∨ r)

Predicate Logic

Predicate	A statement involving variables. P(x) is the predicate P at x.
Quantifiers	Symbols that express the extent to which a predicate is true over a range of elements.
Universal Quantifier (∀)	∀x P(x) means P(x) is true for all x in the domain.
Existential Quantifier (∃)	∃x P(x) means there exists at least one x in the domain for which P(x) is true.
Negation of Quantifiers	¬(∀x P(x)) ≡ ∃x ¬P(x) ¬(∃x P(x)) ≡ ∀x ¬P(x)

Relations

Basic Definitions

Relation	A subset of A × B, where A and B are sets. Represents a relationship between elements of A and B.
Binary Relation	A relation from a set A to itself (a subset of A × A).
Reflexive Relation	A relation R on A is reflexive if (a, a) ∈ R for all a ∈ A.
Symmetric Relation	A relation R on A is symmetric if (a, b) ∈ R implies (b, a) ∈ R.
Transitive Relation	A relation R on A is transitive if (a, b) ∈ R and (b, c) ∈ R implies (a, c) ∈ R.
Equivalence Relation	A relation that is reflexive, symmetric, and transitive.

Properties of Relations

Irreflexive: A relation R on A is irreflexive if (a, a) ∉ R for all a ∈ A.

Antisymmetric: A relation R on A is antisymmetric if (a, b) ∈ R and (b, a) ∈ R implies a = b.

Partial Order: A relation that is reflexive, antisymmetric, and transitive.

Total Order: A partial order where for every a, b ∈ A, either (a, b) ∈ R or (b, a) ∈ R.

Closures of Relations

Reflexive Closure	The smallest reflexive relation containing R. Add (a, a) for all a not already in the relation.
Symmetric Closure	The smallest symmetric relation containing R. If (a, b) ∈ R, add (b, a) to the relation.
Transitive Closure	The smallest transitive relation containing R. Computed using Warshall's Algorithm.

Graph Theory

Basic Definitions

Graph	A pair G = (V, E) where V is a set of vertices and E is a set of edges connecting these vertices.
Directed Graph (Digraph)	A graph where edges have a direction. Edges are ordered pairs (u, v).
Undirected Graph	A graph where edges have no direction. Edges are unordered pairs {u, v}.
Adjacent Vertices	Two vertices are adjacent if they are connected by an edge.
Degree of a Vertex	The number of edges incident to the vertex. In digraphs, indegree is the number of incoming edges, and outdegree is the number of outgoing edges.
Path	A sequence of vertices connected by edges.

Graph Properties

Connected Graph: A graph where there is a path between every pair of vertices.

Complete Graph (Kn): A graph where every pair of distinct vertices is connected by an edge.

Cycle: A path that starts and ends at the same vertex.

Tree: A connected graph with no cycles.

Bipartite Graph: A graph whose vertices can be divided into two disjoint sets U and V such that every edge connects a vertex in U to one in V.

Planar Graph: A graph that can be drawn in the plane without any edges crossing.

Graph Representations

Adjacency Matrix	A matrix representing the graph’s connections. A[i][j] = 1 if there is an edge from vertex i to vertex j, and 0 otherwise.
Adjacency List	A list of adjacent vertices for each vertex in the graph.