Algbra 1 final

A comprehensive cheat sheet covering essential Algebra I concepts, formulas, and techniques. This guide includes data representation, linear equations, functions, exponential functions, and quadratics.

Data Representation and Analysis

Data Representations

Dot Plot:
Simple way to represent data, each dot represents a single observation.

Histogram:
Groups data into bins and displays the frequency of each bin as a bar. No spaces between bars.

Box and Whisker Plot:
Displays the five-number summary: minimum, first quartile (Q1), median (Q2), third quartile (Q3), and maximum.

Useful for identifying the spread and skewness of the data.

Measures of Central Tendency and Spread

Mean:
Average of all data points. Sum of values divided by the number of values.
`Mean = (sum of x) / n`

Median:
Middle value when data is ordered. If there are two middle values, average them.

Mode:
Value that appears most frequently in the data set.

Range:
Difference between the maximum and minimum values.
`Range = Max - Min`

Standard Deviation:
Measures the spread of data around the mean. A lower standard deviation indicates data points are closer to the mean.

IQR (Interquartile Range):
Difference between the third quartile (Q3) and the first quartile (Q1).
`IQR = Q3 - Q1`

Outliers

Outliers:
Data points that are significantly different from other data points in the set.
Can be identified using the 1.5 * IQR rule:
Lower Bound = Q1 - 1.5 * IQR
Upper Bound = Q3 + 1.5 * IQR
Values outside these bounds are considered outliers.

Linear Equations, Inequalities, and Systems

Linear Equations

Standard Form:
`Ax + By = C`

Slope-Intercept Form:
`y = mx + b`, where `m` is the slope and `b` is the y-intercept.

Point-Slope Form:
`y - y1 = m(x - x1)`, where `m` is the slope and `(x1, y1)` is a point on the line.

Solving Linear Equations:
Use inverse operations to isolate the variable. Simplify both sides before isolating the variable.

Systems of Equations

Substitution Method:
Solve one equation for one variable and substitute that expression into the other equation.

Elimination Method:
Add or subtract multiples of the equations to eliminate one variable.

Applications:
Mixture problems, wind/current problems, age problems, coin problems, perimeter problems.

Solving Systems:
Find the values of the variables that satisfy all equations in the system. Represented as an ordered pair (x, y).

Linear Inequalities

Graphing Linear Inequalities:
Graph the boundary line (dashed for < or >, solid for ≤ or ≥). Shade the region that satisfies the inequality.

Systems of Inequalities:
Graph each inequality and find the overlapping shaded region, which represents the solution set.

Linear Programming:
Optimize an objective function subject to constraints. Graph the constraints and find the feasible region. The optimal solution occurs at a vertex of the feasible region.

Example:
`y > 2x + 1`

Functions and Function Notation

Function Basics

Function Notation:
`f(x)` represents the value of the function f at x.

Writing Functions:
Express the relationship between input (x) and output (f(x)).

Tables:
Represent function values in a table format with input and output values.

Graphing:
Plot points (x, f(x)) on a coordinate plane to visualize the function.

Types of Functions

Absolute Value Function:
`f(x) = |x|`
V-shaped graph with vertex at (0,0).

Piecewise Functions:
Defined by different expressions over different intervals of the domain.

Example:

```
f(x) = { x^2, x < 0
       { x + 1, x >= 0
```

Absolute Value as Piecewise:
`|x| = { x, x >= 0 {-x, x < 0`

Example:
`f(x) = |x-2|`

Domain and Range

Domain:
Set of all possible input values (x) for which the function is defined.

Range:
Set of all possible output values (f(x)) of the function.

Set Builder Notation:
`{x | condition}`
Example: `{x | x > 0}`

Interval Notation:
Use brackets [ ] for inclusive endpoints and parentheses ( ) for exclusive endpoints.
Example: `(0, ∞)`

Inverse Functions

Inverse Function:
If `f(x)` maps x to y, then `f⁻¹(y)` maps y back to x.
To find the inverse, swap x and y and solve for y.

Exponential Functions

Exponential Function Basics

General Form:
`f(x) = a * b^x`
where `a` is the initial value and `b` is the base (growth/decay factor).

Exponential Growth:
`b > 1`
The function increases as x increases.

Exponential Decay:
`0 < b < 1`
The function decreases as x increases.

Solving Exponential Equations:
Use logarithms or rewrite with a common base.

Exponent Rules

Exponent Rules Summary:

- `x^m * x^n = x^(m+n)`
- `(x^m) / (x^n) = x^(m-n)`
- `(x^m)^n = x^(m*n)`
- `(xy)^n = x^n * y^n`
- `(x/y)^n = x^n / y^n`
- `x^0 = 1`
- `x^(-n) = 1 / x^n`

Compound Interest

Compound Interest Formula:
`A = P(1 + r/n)^(nt)`
where:
`A` = the future value of the investment/loan, including interest
`P` = the principal investment amount (the initial deposit or loan amount)
`r` = the annual interest rate (as a decimal)
`n` = the number of times that interest is compounded per year
`t` = the number of years the money is invested or borrowed for

Percent of Change

Percent of Change Formula:
`Percent Change = ((New Value - Old Value) / Old Value) * 100`

- Positive result indicates percent increase.
- Negative result indicates percent decrease.

Quadratic Functions

Factoring

Factoring Quadratics:
Expressing a quadratic expression as a product of two binomials.
Example: `x^2 + 5x + 6 = (x + 2)(x + 3)`

Zero Product Property:
If `ab = 0`, then `a = 0` or `b = 0`.
Used to solve factored quadratic equations.

Difference of Squares:
`a^2 - b^2 = (a + b)(a - b)`

Perfect Square Trinomial:
`a^2 + 2ab + b^2 = (a + b)^2`
`a^2 - 2ab + b^2 = (a - b)^2`

Quadratic Forms and Graphs

Standard Form:
`f(x) = ax^2 + bx + c`

Factored Form:
`f(x) = a(x - r1)(x - r2)`, where `r1` and `r2` are the roots (x-intercepts).

Vertex Form:
`f(x) = a(x - h)^2 + k`, where `(h, k)` is the vertex.

Graphing Quadratics:
Parabola shape.
Vertex is the minimum (if `a > 0`) or maximum (if `a < 0`) point.

Solving Quadratic Equations

Quadratic Formula:
`x = (-b ± √(b^2 - 4ac)) / (2a)`
Used to find the roots of a quadratic equation in standard form.

Completing the Square:
Transform the quadratic equation into vertex form and solve for x.

Discriminant:
`Δ = b^2 - 4ac`

- If `Δ > 0`, two real solutions.
- If `Δ = 0`, one real solution.
- If `Δ < 0`, no real solutions (two complex solutions).

Geometric Patterns:
Quadratic functions can represent areas and patterns that grow quadratically.