Page 1: Problem Solving & Modeling

Problem Solving Framework

1. Define the Problem: Clearly state the problem, including objectives and constraints.
2. Gather Information: Collect relevant data, research, and identify assumptions.
3. Generate Solutions: Brainstorm potential solutions and evaluate their feasibility.
4. Implement and Test: Choose the best solution, implement it, and test its effectiveness.
5. Evaluate and Iterate: Analyze the results, identify areas for improvement, and refine the solution.

Example: Designing a bridge to withstand specific loads. Defining the problem would involve understanding the required load capacity, environmental conditions, and material constraints.

Modeling Principles

Abstraction: Simplifying a complex system by focusing on essential features and ignoring irrelevant details.	**Example:** Modeling a car's fuel efficiency might involve considering engine size and weight but ignoring the color of the car.
Assumptions: Making informed guesses about aspects of the system that are unknown or too complex to model directly.	**Example:** Assuming air resistance is negligible when modeling the trajectory of a ball thrown at low speeds.
Validation: Ensuring that the model accurately represents the real-world system and produces reasonable results.	**Example:** Comparing the model's predicted bridge deflection under load with actual measurements from a physical prototype.

Types of Models

Physical Models: Tangible representations of a system (e.g., a scale model of a building).
Mathematical Models: Use equations to describe the behavior of a system (e.g., Newton's laws of motion).
Computational Models: Use computer simulations to analyze and predict system behavior (e.g., finite element analysis).

Example: A wind tunnel test using a physical model of an airplane wing. A set of differential equations modeling population growth. A computer simulation predicting weather patterns.

Page 2: Data Analysis & Statistics

Descriptive Statistics

Mean: Average value of a dataset. $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$	**Example:** The mean height of students in a class.
Median: Middle value in a sorted dataset.	**Example:** The median income of households in a city.
Standard Deviation: Measure of data dispersion around the mean. $s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$	**Example:** The standard deviation of test scores indicates the spread of student performance.

Probability Distributions

Normal Distribution: Symmetrical bell-shaped distribution characterized by mean (μ) and standard deviation (σ).
Uniform Distribution: All values within a range are equally likely.
Binomial Distribution: Probability of success in a sequence of independent trials.

Example: Heights and weights often follow a normal distribution. Rolling a fair die follows a uniform distribution. The number of heads in 10 coin flips follows a binomial distribution.

Error Analysis

Systematic Error: Consistent bias in measurements.	**Example:** A miscalibrated measuring instrument.
Random Error: Unpredictable fluctuations in measurements.	**Example:** Measurement noise due to environmental factors.
Accuracy: Closeness of a measurement to the true value.	**Precision:** Repeatability of a measurement.